



# Inside the Atoms: Ranking on a Network of Networks

Jingchao Ni<sup>1</sup>, Hanghang Tong<sup>2</sup>, Wei Fan<sup>3</sup>, and Xiang Zhang<sup>1</sup> <sup>1</sup>Department of Electrical Engineering and Computer Science, Case Western Reserve University <sup>2</sup>School of Computing, Informatics, Decision Systems Engineering, Arizona State University <sup>3</sup>Huawei Noahs Ark Lab

### **Network of Networks**

Motivation: Network is an important and popular data model since real-world data are naturally networks, e.g., web network, social network, biological network, etc. However, networks are not independent. For example, a co-author network of data mining area is highly related to a co-author network of database area. In fact, networks themselves form a network. We do not want to ignore this high level network since it helps us learn more about the data. We call such structure as a **Network of Networks (NoN)**, such as research area network of co-author networks, disease similarity network of protein interaction networks, etc.  $\widetilde{\mathbf{A}} = \begin{bmatrix} \widetilde{\mathbf{A}}_1 & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \widetilde{\mathbf{A}}_g \end{bmatrix}$ 

#### Research area network of co-author networks Disease network of protein interaction networks

 $\mathbf{r} = \begin{vmatrix} \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_g \end{vmatrix}$  $\mathbf{e} = \begin{vmatrix} \mathbf{e}_1 \\ \vdots \end{vmatrix}$ X: A normalized Laplacian matrix of cross network links between common nodes

Matrix form objective function

 $\mathbf{J}(\mathbf{r}) = c\mathbf{r}'(\mathbf{I}_n - \widetilde{\mathbf{A}})\mathbf{r} + (1 - c)\|\mathbf{r} - \mathbf{e}\|_{E}^{2} + 2a\mathbf{r}'\mathbf{X}\mathbf{r}$ 

### Solution to CrossRank

**RWR-like update rule**  $\mathbf{r} = \left(\frac{c}{1+2a}\widetilde{\mathbf{A}} + \frac{2a}{1+2a}\widetilde{\mathbf{Y}}\right)\mathbf{r} + \frac{1-c}{1+2a}\mathbf{e}$ **Property:** Eigenvalues of the transition matrix are between -1 and 1 $\geq$  It converges to the global minimum of the objective function



Examples of NoN. The main network is represented by dashed nodes and edges. The domainspecific networks are represented by solid nodes and edges.

*Definition.* Network of Networks (NoN). Given a  $g \times g$  main network G, a set of g domain-specific networks  $\mathcal{A} = \{A_1, \dots, A_g\}$  and a one-to-one mapping function  $\theta$ , which maps each node in the main network G to a domain-specific network, a **Network of Networks (NoN)** is defined as the triplet



Target domain-

Database

Source domain

specific network

Different people have different interests in nodes of a specific network network. They may want to find top-k "similar" nodes in a network w.r.t. a query node. This is a ranking problem with query node.

NoN allows us to query a node in a source domainspecific network and retrieve top-k "similar" nodes from a target domain-specific network:

> Which bioinformatics researchers will collaborate with the data mining researcher Jon?

CrossQuery-Basic: RWR-like update rule allows us to apply existing scalable algorithms for RWR.

**CrossQuery-Fast:** 1. Extract relevant subnetwork w.r.t. main nodes representing source and target Main network domain-specific networks from the main network; 2.

 $\mathcal{R} = \langle \mathbf{G}, \mathcal{A}, \theta \rangle$ . Nodes in the main network are referred to as main nodes, nodes in the domainspecific networks are called *domain nodes*. Each main node represents a domain-specific network through the mapping function  $\theta$ . In addition, we represent the nodes in each domain-specific network as  $\mathcal{V}_i$  (i = 1, ..., g). We define  $I_{i,i}$  as the common nodes between  $\mathbf{A}_i$  and  $\mathbf{A}_i$ , i.e.,  $I_{i,i} = \mathcal{V}_i \cap \mathcal{V}_i$ .

### CrossRank

Given a network, ranking is an important task. People want to quickly identify important nodes (e.g., users, genes, etc.) from a network with thousands or millions of nodes.

NoN allows us to rank nodes in broader view:

> Who is more important in data mining area? Jon or Jim? If we consider data mining only? > Who is more important in data mining area? Jon or

Jim? If we consider all highly related areas to data mining?

We propose a regularized optimization model to rank domain nodes w.r.t. the main network, i.e., minimizing:

## $\mathbf{J}(\mathbf{r}_1,\ldots,\mathbf{r}_g) = c \sum_{i=1}^{\infty} \mathbf{r}_i' (\mathbf{I}_{n_i} - \widetilde{\mathbf{A}}_i) \mathbf{r}_i + (1-c) \sum_{i=1}^{\infty} \|\mathbf{r}_i - \mathbf{e}_i\|_{i=1}^{2}$





Ínfo. Retrie

Relevant subnetwork

#### Experiments

#### **Co-author NoN**



#### **Protein Interaction NoN**

Disease Similarity Network

**Disease Similarity Network** 



Cross-area co-authorship

Top ranked authors in the database area when varying a

Data Minin

Info. Retrie

Database

Machine Learnir

